SZEGŐ-WIDOM ASYMPTOTICS OF CHEBYSHEV POLYNOMIALS
ON CIRCULAR ARCS

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Abstract

We review the main results of the seminal paper of Widom [2] on asymptotics of orthogonal and Chebyshev polynomials associated with a set $E$ (i.e., the monic polynomials of degree at most $n$ that minimize the sup-norm $\|T_n\|_E$), where $E$ is a system of Jordan regions and arcs. Thiran and Detaille [1], considered the Chebyshev polynomials $T_n$ on a circular arc $A_\alpha$ and managed to find an explicit formula for the asymptotics of the extremal value $\|T_n\|_{A_\alpha}$, disproving a conjecture of Widom stated in the aforementioned paper. We give the Szegő-Widom asymptotics of the domain $\mathbb{C} \setminus A_\alpha$ explicitly, i.e., the limit of the properly normalized extremal functions $T_n$. Moreover, we solve a similar problem with respect to the upper envelope of a family of polynomials uniformly bounded on $A_\alpha$. Our computations show that in the proper normalization the limit of the upper envelope is the diagonal of a reproducing kernel of a certain Hilbert space of analytic functions.

References


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